Voter Information and Cues in Direct Legislation Settings

Frederick J. Boehmke\textsuperscript{1}

Department of Political Science

University of Iowa

John W. Patty\textsuperscript{2}

Department of Social and Decision Sciences

Carnegie Mellon University

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\textsuperscript{2}Corresponding Author: jwpatty@flatline.hss.cmu.edu
Abstract

This paper develops a model which joins voters’ behavior in initiative and legislative elections with the legislative treatment of interest groups. The model provides several insights into the inferences voters should draw concerning the nature of proposed initiatives. By observing the initiative, all voters should be aware that the legislature has chosen not to enact the proposed legislation. We show that this fact, combined with the different incentives facing the legislature when dealing with large and small interest groups, implies that the expected value of an initiative is strictly decreasing in the size of the sponsoring group. Furthermore, not only does this monotonicity hold for both members and nonmembers of the sponsoring group, but the benefits offered by an initiative to members and nonmembers are negatively correlated.
1 Introduction

Over the past 30 years, the initiative process has played an increasingly important role in the formation of public policy in several states. One of the biggest concerns about substituting citizen legislators for elected legislators is whether voters are sufficiently informed to make the correct decision when they vote. While concerns about the informedness of voters is not a new topic, studying it in the context of initiatives and referendums is still much rarer than in the context of candidate elections. The direct legislation setting has some similarities and dissimilarities to candidate elections. In this paper we focus on one of the dissimilarities: how should voters react to the information associated with every ballot question, namely that the legislature chose not to implement the proposed policy change?

To study the interaction between voters’ decisions and the legislature’s actions, we must also consider the role of interest groups. Interest groups play a pivotal role in placing initiatives on the ballot and rallying electoral support for their passage. This paper provides insight into the interplay between the interest group proposing an initiative and the inferences a voter should draw regarding the effect of the proposed policy on his or her well-being. We show that the initiative process is characterized by adverse selection. Furthermore, we show that this adverse selection is worse for large interest groups: the expected benefit of an initiative is negatively related to the size of the sponsoring group, for both its members and nonmembers.

1.1 Related Literature

The seminal model of direct legislation is due to Romer and Rosenthal (1978). Romer and Rosenthal, among several others (e.g., Morton (1988) and Banks (1990), (1993)), examine the effects of
direct legislation in what has become known as the setter model. In this framework, a proposer makes a take-it-or-leave-it offer to a group of voters. The general finding is that the proposer extracts rents from the voters (e.g. a budget-maximizing bureaucrat sets expenditures higher than the median voter’s most preferred level). This results from the power inherent in controlling the agenda. Our model is related to the setter model, but we do not allow the setter to choose her policy. Instead, we are interested in what voters can infer about proposed policies as a result of their proposal as initiative.

Many authors have written about the informedness of voters (see, for example, Popkin (1991)). The general finding (both theoretical and empirical) is that few voters have an incentive to gather extensive information regarding proposed initiatives. This is due to the fact that, in general, no voter’s vote on an initiative will be pivotal: it is incredibly unlikely that the outcome of the election will be a direct consequence of any voter’s vote choice. Nevertheless, it is obvious that a voter should take into account any costless information regarding a proposed initiative. This paper examines one very simple piece of information that voters may use to form their beliefs regarding the expected value of an initiative: the size of the interest group proposing the initiative.

The direct legislation setting provides an important and different setting in which to explore the consequences of rational voter ignorance. By shifting from representative to direct democracy, the representative, and any information associated with her, is removed from the policy-making process. Secondly, the lack of traditional cues such as party affiliation and candidate personality and the relative complexity of ballot measures can leave voters with little, if any, information upon which to base their decisions. Cronin (1989) summarizes the situation that voters, especially the less educated and less well off, face in ballot issue elections: “…‘information costs’ …are
generally even higher than in candidate elections. Legal and technical language on ballot issues sometimes causes confusion, and the absence of party labels usually attached to candidates denies a majority of these voters a familiar cue” (Cronin (1989), p. 67) He reports that over a quarter of voters say that they have difficulty choosing wisely on ballot issues.

Magleby (1984) reaches even more dramatic conclusions, describing voters as facing an “informational vacuum,” leading to votes that are essentially “electoral roulette.” By realizing that representatives are not completely out of the equation, however, we study how voters can increase their level of information using two simple pieces of information: (1) the legislature chose not to enact the policy contained in the initiative itself and (2) the number of members in the group.

Other work has also studied the types of cues that are available in initiative voting. Lupia (1994) conducted an exit poll to gather information about California voters’ information concerning five insurance industry related propositions on the ballot in 1988. Three of these propositions were sponsored by the insurance industry, one by a trial lawyers’ association, and one by a consumer activist group. Regardless of a voter’s level of technical information about the five initiatives, Lupia found that knowledge of the sponsoring group’s identity significantly affected the probability of supporting any of them. For example, voters who had low information were at least 18% less likely to support any of the insurance industry initiatives if they were aware of the industry’s support of them, whereas they were 68% more likely to support the activist group’s Proposition 103. This may be why in advertisements sponsored by the insurance industry and the trial lawyers’ association, “the identity of the sponsor was as well hidden as the law would allow” (Lupia (1994), 94).

In separate work, Lupia (1992) also provides a theoretical model of voter behavior in direct legislation elections. Using a spatial model he assumes voters have perfect information about the
status quo, but do not know the location of the initiative or the ideal of the group proposing it. In the basic model, the fact that the interest group is willing to make a costly proposal provides voters with valuable information. In one variation on his model, Lupia assumes that voters use “elite” endorsements to make more accurate inferences regarding the location of the proposed initiative. While our model and that developed by Lupia are not mutually exclusive — voters may use both types of information — our model utilizes a novel cue for voter behavior that exists in every initiative campaign. We discuss the possibility of extending our model to include third-party endorsements in Section 5.

As far as we can tell, our model generates a novel insight into the interplay between legislative action and the initiative process. In equilibrium, the fact that the legislature decided not to enact a proposal yields information about the initiative. For example, if a large group fails to get legislative approval for its policy, voters can infer that the initiative must have negative effects for nonmembers who, therefore, may be more likely to vote against it. In our model, if there were positive benefits for everyone the legislature would not have passed up the opportunity to enact the group’s proposal as legislation. Specifically, we show that the expected utility of the initiative to non-group members is decreasing in the group’s size, which decreases the probability of nonmembers voting for the initiative. The result is also illustrated by performing a series of simulations.

2 The Model

There are four classes of individuals in our model: the voters in the group (i.e., the “members”), all other voters (the “nonmembers”, who are assumed to have preferences that may differ from those of the members, but are identical otherwise), the incumbent legislator, and the challenger legislator
(who has no actions available to her). The legislator perfectly observes the group’s proposal and may either enact or reject it. If the proposal is enacted, then the voters vote only between the challenger and the incumbent legislator. In this case the voters will have observed the value of the proposal after the legislator enacted it, eliminating any uncertainty as to the true value of the proposal to each voter. We also assume that, if the legislator enacts the proposal, it is repealed if the challenger wins the election, although all we really need is that any given voter is less likely to vote for the incumbent legislator if the policy enacted hurts him or her. If the proposal is not enacted by the legislator, then the voters must vote to either accept or reject the proposal as an initiative as well as between the challenger and the incumbent legislator, whose policy position is the original status quo. In this case, the voters are not aware of the true value of the proposal when they cast their vote since the proposal has not been implemented yet. They are aware of their own preferences, the number of other voters in the group, and the fact that the legislator decided to reject the proposal.

Formally, a voter’s preferences are the sum of three possible components – a privately known, idiosyncratic preference for the incumbent politician’s policies \( t^1_i \), a similar private information preference for the group’s proposal \( t^2_i \), and a preference shock arising from passage of the proposal that affects group members and nonmembers differentially \( y_i \). The game tree is given in Figure 1.

Our theory makes a number of assumptions, most of them technical in nature but a few that are substantive. First, we assume that voters are strategic and have privately known, idiosyncratic preferences with respect to passage of the proposal and the winner of the legislative election. These preferences lead to “probabilistic” behavior, but our model assumes full rationality: the players’
Figure 1: The Extensive Game Form
strategies constitute a *Perfect Bayesian Equilibrium in weakly undominated strategies*. Examining such perfect Bayesian equilibria essentially restricts voters’ beliefs to be correct, given the strategy of the legislator. Restricting attention to equilibria in which voters use weakly undominated strategies ensures that voter behavior is not pathological.

Second, we assume that the incumbent legislator chooses to enact or reject the proposal so as to maximize the number of votes she receives in her reelection bid. Third, as noted above, we assume that the proposal, if enacted by the legislator, is rescinded if the challenger defeats the incumbent. Finally, we assume that the payoff a voter receives from a proposal passed as an initiative is independent of who wins the legislative election. We now move to the predictions of our theory. We relegate the technical development of the model and the proofs of our results to Appendix A.

### 3 Results

Our theory provides several insights into the interplay between legislative action and initiatives. In particular, our theory highlights the fact that, absent the effect of district-specific representation, an office-seeking legislator has no *a priori* incentive to have legislation passed through the initiative process. A relatively simple thought experiment indicates that, in a world with one legislator (i.e., one district), complete information, and neutral treatment of policies passed by the legislature and those passed by initiative, no legislation should be enacted through the initiative process.

Our theory predicts that the expected value of an initiative is decreasing in the size of the group supporting it (this result is proven formally as Proposition 6 in Appendix A).
**Result 1** *The expected value of an initiative is decreasing in the size of the sponsoring group.*

The intuition behind this is simple. If the group proposes it, it must be individually rational for them to do so - the group’s members must be expecting to benefit from the proposal if enacted. Second, an office-seeking incumbent legislator would enact any proposal which helped the group and did not systematically hurt the nonmembers. Third, the legislator should be willing to impose some costs on nonmembers if the benefits to the group’s membership are large enough. Finally, the acceptable size of such costs are increasing in the size of the group. In sum, the only reason a proposal is proposed by a large interest group and *not* passed by the legislature is that, while the proposal offers benefits to the group, it imposes severe costs on nonmembers. Note that this argument also applies to group members: for the legislature not to pass the proposal, the benefits for group members must not have warranted the costs imposed on non-members (though at least they are expected to be positive for members).

By the same logic, we can relate the expected benefits for members and non-members. The greater the benefits offered by an initiative to the proposing group’s members, the worse the costs imposed on nonmembers must be for it not to have been previously enacted by the legislature (this result is proven formally as Proposition 7 in Appendix A).

**Result 2** *The expected value of an initiative to nonmembers is decreasing in the size of the benefits offered by the initiative to group members.*

Based on our model, then, initiative voters should therefore utilize three pieces of information. First, and most importantly, they should utilize group membership as their first cue. Group members have positive expected utility from the initiative whereas non-members have negative expected utility. Second, they should consider the size of the proposing group. The large the group,
the worse off the initiative would be expected to make them if passed.

Our results are straight-forward, but their implications are far-reaching. Result 1 implies that size may indeed be a curse for groups seeking to achieve policy outcomes through the initiative process. The model makes no prediction about the relative success of large and small groups in initiative elections, as this depends on the distribution of voters’ types and, of course, larger groups do have an advantage from the simple fact that more of the voters are \textit{in} the group.

Result 2 implies that, in an initiative campaign, sponsors of the initiative have an incentive to disguise how much they expect to benefit from enactment of the proposed policy. Indeed, Result 2 indicates a possible incentive for groups to campaign against their own initiatives (assuming that the sponsoring group can somehow hide their sponsorship of the initiative).

Finally, our model makes a general point regarding the nature of initiative legislation: in equilibrium, initiatives should be polarizing in that they make some voters better off at the expense of others. Otherwise, the initiative would either not have been proposed or it would have been preemptively enacted by the legislature.

4 Simulation Results

In this section of the paper, we explore the relationship between group size and various outcomes of the model by performing a series of computer simulations. These simulations show how changes in the size of the interest group affect both members and non-members’ utilities, depending on whether the legislature implements the group’s proposal.

To perform the simulations, we fix all but the stochastic parameters of of the formal model and then generate a randomly drawn proposal. The group decides whether to offer the proposal to the
legislature, which then accepts or rejects the proposal. If it is rejected, the group can then submit the proposal to the voters as an initiative, where it is passed or failed. Once the game ends, the outcome is noted and utilities are received and recorded. The game is then repeated 1000 times for each group size with new random draws each time. After all 1000 draws are completed we increase the size of the group and start over.

The parameters that are held fixed are as follows. Population size, $N$ is set at 1001, out of which $G$ individuals will be in the group and $N - G$ will be non-members. Proposals are drawn from a uniform distribution: $y \sim U[-1, 1]^2$ and the random utility components $t_{1i}^1$, for the incumbent’s re-election, and $t_{1i}^2$, for the proposal, are independently drawn from a standard normal distribution.

We start with a group size of 1 and increment it by one until it reaches 500, with 1000 repetitions for each value. Since the model excludes cases where it is not individually rational for the group to offer the proposal either to the legislature or as an initiative, these cases are not allowed in the simulations. The results are presented in Figures 2, 3, 4, and 5.

Figure 2 shows the frequencies of certain outcomes, namely how often the incumbent implements the proposal, how often the incumbent wins re-election and how often initiatives on the ballot receive a majority of votes. As the model predicts, as the group gets larger the incumbent becomes more likely to implement its proposals since a greater number of individuals will benefit. The opportunity to implement the group’s proposal is beneficial for the incumbent, who is able to win re-election about 74% of the time even when the group has only one member. This advantage increases as the group gets larger and reaches 85% when the group comprises half the total population. The third line shows how often initiatives that make the ballot pass. Note there is an initiative on the ballot whenever the incumbent rejects the proposal since we limit our analysis to
proposals that benefit the group. No initiatives are passed until the group reaches 346 members, when the proportion that pass is less than 1% (1 out of 371 initiatives passed, or 0.27%). Larger group sizes lead to increased probability of passage for initiatives, but the proportion of initiatives passing never exceeds 15%.4

Figure 2 here.

Figure 3 shows the expected utility for group members and non-members from initiatives (i.e., proposals that the incumbent did not enact). The incumbent’s rejection of the proposal, coupled with the fact that the group does not propose something that hurts members, implies that non-member utility must be less than zero and it gets increasingly lower as \( G \) increases. While members always benefit from an initiative, as the group size increases, the expected benefit of initiatives decreases not only for nonmembers, but for members as well.

Figure 3 here.

The trade-off that the incumbent makes is also demonstrated by the average utilities obtained when she accepts proposals, displayed in Figure 4. Member utility is relatively constant at 0.5, though it does increase slightly with the size of the group. Non-member utility starts out the same as member utility, but it has a pronounced decline as \( G \) increases. When the group has about 300 members, average non-member utility is around 0.4 and when group size reaches 500, it has dropped to 0.25. This is a consequence of the incumbent trading off more member votes for less non-member votes: she is willing to impose greater harm on non-members when the total benefit from group members is greater (the group is larger). Thus in all cases, our main result is illustrated: as group size increases, average non-member utility decreases. Non-members should use this information when voting on initiatives.
Figure 5 displays the correlations between members and nonmember average utilities as a function of group size. As predicted by Result 2, these correlations decrease (i.e., become more negative) as the group size increases. Furthermore, the correlations generally become stronger in the presence of initiatives than when the proposal is enacted by the legislature. Again, this results from the fact that the legislature must take into account both members and nonmembers when deciding whether or not to enact a proposal. Consider rejected proposals. When the group’s size is zero, there should not be any correlation between member and non-member utility since any proposal with negative non-member utility will immediately be rejected. As group size increases, the legislature is willing to accept some costs for non-members to benefit members. So when presented with a proposal that offers large benefits to members, it must impose significant costs on non-members to be rejected, producing the negative correlation. As the group’s size increases the benefits are felt by a larger portion of the population and the costs by a smaller portion, allowing the legislator to impose greater costs on non-members for a given increase in member benefits. This leads to the stronger relationship between member and non-member utility as group size increases and demonstrates that non-members should be more wary of initiatives proposed by large groups.

5 Extensions

In this section we briefly mention some possible extensions to the model. In particular, we discuss the possibility of allowing elites to offer cues to voters and the implications of relaxing the assumption that the legislature can preemptively enact a proposal.
5.1 Endorsements

Our model could be extended to allow for endorsements — costless, nonbinding signals — by interest groups and/or “elites”, as in Lupia (1992). Underlying such an extension would be an assumption that elites could obtain possibly costly signals about the true value of proposed legislation and initiatives. In such a model, the number of elite endorsements would affect voter behavior in equilibrium. Such an extension could allow for asymmetric information between the proposer and the legislature.

Elite endorsements (or cues in general) could also be recast as attempts to define group membership. If voters can be convinced to identify with the group than they will expect to receive the greater level of benefits associated with group affiliation. The attempts at persuasion will then focus on identifying group members. While this may be unsuccessful with well-defined groups that require membership fees and offer selective benefits, in many cases it may not be obvious which voters are in the group (policies to benefit the elderly or the working class, for example). By making group membership endogenous, we could explore this issue within a framework much like the one presented here.

5.2 Preemptive Passage Power

One assumption of our model is that initiatives have been considered by the legislature before being placed on the ballot as initiatives. What are the incentives for interest groups to deviate from this, that is, to place initiatives directly on the ballot without consideration by the legislature? In equilibrium, the voters will infer that any initiative was either rejected by the legislature or not submitted to the legislature at all. The first case is dealt with in our earlier analysis. However, no
group would bypass the legislature and submit it to the voters as an initiative if the legislature would have enacted the proposal. Therefore, the voters’ inference would remain the same regarding the true value of an initiative as a function of group size and the fact it was not enacted by the legislature.

6 Discussion and Conclusions

There has been a great deal of debate over the implications of using the initiative process to enact public policy. Claims about voter ignorance being substituted for the legislative process, which is designed to debate policy and make the compromises necessary for successful policy are frequent (Magleby (1984); Schrag (1998); Broder (2000)) Our model makes an obvious statement about the first of these objections: the information needed for voters to make an informed vote may not be as difficult to come by as current opinion suggests.

An important implication and assumption of our model is that voters can identify the group that is sponsoring the initiative. In many cases, as Lupia (1994) notes, groups do their best to hide their identity and create temporary groups with populist-sounding names (“Citizens for...”) to manipulate the concept of membership. Our findings suggest one reason that disclosure requirement may be important. By requiring groups to reveal their identity in advertising, disclosure laws would allow voters are to learn about the size of the group involved and the potential benefits they will receive under the proposed legislation.

Studying the requirements for advertising in initiative campaigns across states may provide an empirical test of the model: in states where groups can hide their identity, voters will not be able to use group size as a cue. We might expect that the effect of group size will be dampened or
non-existent in these states. In states where the group is required to reveal its identity, we would expect that nonmembers would be more likely to support proposals by smaller groups.

Note that the model does not predict that the initiative is less likely to pass. While increasing group size decreases the probability of nonmembers voting for the initiative, adding people to the group increases the probability that they will vote for it. The net effect is therefore not clear: the overall chance of successfully passing the proposal may go up or down. Interestingly, however, it is only for the affiliation switchers that the expected utility goes up since, as Result 1 states, the expected utility of an initiative to both group members and nonmembers is inversely related to the group’s size.

The issue of identification notwithstanding, one of the appealing aspects of our model is that the membership of many groups is a fixed quantity. Unlike cues taken from elites, it can be verified on the part of voters. Further, opponents of ballot measures generally like to point out who are really behind them, so many voters will have the opportunity to assess how the fact that the group has been rejected by the legislature should influence their vote.
Notes

1 We assume that all possible proposals make the group better off. The model is trivial whenever the proposal is not offered, so this assumption is made only for clarity and imposes no substantive restriction on our results.

2 We conjecture that our results would remain qualitatively unchanged if we assumed that the incumbent seeks to maximize her probability of victory.

3 We do not count these as repetitions — the results presented are based on 1000 cases where the proposal offers group members positive utility.

4 The maximum success rate for initiatives was 10.3%, which was observed with a group size of 477.

5 This happens since the legislature will implement the proposal for sure whereas voters implement it with some probability strictly less than zero.
References


A Formal Presentation of the Model

A.1 Assumptions

The incumbent legislator is denoted by $L$ and the challenger by $C$. In an abuse of notation, the set of voters is denoted by $N = \{1, \ldots, N\}$, with $N$ odd, and is partitioned into two subsets, $G$ and $G^C$, corresponding to the membership of the group and its complement. The voters are ordered so that $i \leq G$ implies that voter $i$ is a member of $G$ and $i > G$ implies that $i \in G^C$.

The proposer is endowed with a proposal, $y \in \mathbb{R}^N$, where the $i^{th}$ component, $y_i$, represents the utility gained by voter $i$ if the proposal is enacted by either the legislator or by the electorate as an initiative. We assume that $y_i = y_j$ for all $i, j \leq G$ and $y_k = y_l$ for all $k, l > G$. This captures the idea that the proposal represents a group-specific policy change. We also assume that $y_i > 0$ for all $i \leq G$, implying that any proposed policy offers positive expected benefits to group members, as any other type of policy would not be proposed by the group. It is common knowledge that $y$ is randomly drawn according to a probability measure $\rho : \mathcal{B}(\mathbb{R}^n) \rightarrow [0, 1]$, where $\mathcal{B}(\mathbb{R}^n)$ denotes the set of Borel subsets of $\mathbb{R}^n$. We denote the support of $\rho$ by $Y \subset \{y \in \mathbb{R}^N : y_i = y_j > 0 \forall i, j \leq G; y_k = y_l \forall k, l > G\}$.

The game tree is given in Figure 1. The incumbent legislator, $L$, can either enact the proposal or reject it. Regardless of her decision, she must face the challenger, $C$, in an election. If the proposal is rejected by the legislator, then the voters vote whether or not to enact it as an initiative. If the legislator decides to enact the proposal and the challenger is elected in the legislative election, then the proposal is assumed to be rescinded. If a majority of voters vote in favor of the proposal then it is enacted. We refer to as a proposal as passing if it is enacted either by the legislature or as an
We assume that the voters have incomplete information about the proposal when voting on it as an initiative. Since the game tree is common knowledge, the voters are aware that the legislator did not enact any proposal that is proposed as an initiative but uncertain as to their payoff if it passes. On the other hand, if the legislator enacts a proposal submitted by the group then the proposal’s value to both members and nonmembers is realized prior to the election and, hence, is used by the voters when evaluating the expected utility of keeping the incumbent in office.

A.2 The Voters

For each $i \in N$, let $T = \mathbb{R}^2$ denote the type space of voter $i$. The first dimension of any voter $i$’s type, $t^1_i$, represents an additive utility shock resulting from a victory by the incumbent legislator, $L$, while the second dimension, $t^2_i$, represents an additive utility shock resulting from the passage of the proposal. We assume that voters’ types are independently and identically distributed. In particular, for all $i \in N$, the type of voter $i$, $t_i \in T$, is private information and distributed according to a probability measure $\phi$, possessing full support on $T$. We assume that

1. $\phi$ is twice continuously differentiable and

2. the two dimensions of $t_i$, $t^1_i$ and $t^2_i$, are independently and identically distributed, each with marginal distribution $f$ which is symmetric about zero.

We will denote the product measure on the space of voter type profiles by $\hat{\phi} = \phi^N$. 
We assume the payoff (or utility) function for voter $i$ is given by 

$$
\pi_i = \begin{cases}
    t_1^i & \text{if proposal fails and } L \text{ wins} \\
    0 & \text{if proposal fails and } C \text{ wins} \\
    y_i + t_1^i + t_2^i & \text{if proposal passes and } L \text{ wins} \\
    y_i + t_2^i & \text{if proposal passes by initiative and } C \text{ wins} \\
    t_2^i & \text{if proposal passed by } L \text{ and } C \text{ wins}
\end{cases}
$$

Note that if the incumbent passes the proposal and the challenger wins the legislative election, then the proposal is rescinded. Thus, the incumbent should enact the proposal only if she would prefer to be associated with the proposal than with the status quo.

The set of actions available to voter $i$ depends on the decision of the legislature regarding the proposal. If the legislator enacts the proposal, each voter must vote for either the incumbent or challenger legislator. If the proposal is rejected, then the voters must vote in favor of or against the initiative as well as for either the incumbent or challenger legislator. We do not allow voters to abstain from either election: each voter must vote for either the incumbent legislator or the challenger and either for or against the initiative if the proposal was not enacted by the legislator.

We denote the vote of voter $i$ in the legislative election by $v_i^{L} \in \{0, 1\}$, with $v_i^{L} = 1$ representing a vote in favor of the incumbent and $v_i^{L} = 0$ representing a vote in favor of the challenger. If the legislator rejects the proposal, voter $i$’s vote in the initiative election is denoted by $v_i^{I} \in \{0, 1\}$, with $v_i^{I} = 1$ representing a vote in favor of enacting the initiative and $v_i^{I} = 0$ representing a vote in favor of rejecting the initiative.
A strategy for voter $i$ is a mapping $s_i$ from information sets to lotteries over the appropriate actions. Note that the space of information sets for any voter $i$ is enormous: for any voter $i$ there is an information set for every proposal-type pair, $(y,t_i)$, as well as an additional information set for every type of voter $i$, $t_i$, which is visited by voter $i$ if the legislator rejects the proposal and voter $i$'s type is $t_i$. Thus, the set of information sets for any given voter is $\mathcal{I} = (Y \cup \emptyset) \times T$. We denote the information set of voter $i$ following enactment of a proposal $y$ and realization of type $t_i$ by $\mathcal{I}(y,t_i)$ and the information set of voter $i$ following rejection of any proposal and realization of type $t_i$ by $\mathcal{I}(\emptyset,t_i)$. We assume that $\mathcal{I}$ is endowed with the product topology and its corresponding Borel $\sigma$-algebra. The actions available to voter $i$ at information set $I \in \mathcal{I}$ are represented by a correspondence $\mathcal{A} : \mathcal{I} \rightarrow \{\{0,1\},\{0,1\}^2\}$. Letting $\Delta(\mathcal{A}(\mathcal{I}))$ denote the set of mixed strategies (i.e., lotteries) over the finite set $\mathcal{A}(\mathcal{I})$, we require that the strategy of each voter $i$ be a Borel measurable function, $s_i : \mathcal{I} \rightarrow \Delta(\mathcal{A}(\mathcal{I}))$, with the set of all such Borel measurable functions denoted by $S_i$. The posterior belief of voter $i$ at information set $\mathcal{I}(\emptyset,t_i)$ regarding the true value of the initiative is denoted by $\mu_i(\cdot,t_i) : \mathcal{B}(Y) \rightarrow [0,1]$, where $\mathcal{B}(Y)$ denotes the set of Borel subsets of $Y$. We denote the vector of strategies for all voters $i \in N$ by $s$ and the vector of beliefs for all voters by $\mu$.

We write voter $i$'s strategy as $s_i = (s_1^i, s_2^i, s_3^i)$, with $s_1^i(y,t_i)$ denoting the probability of voting for the incumbent legislator given enactment of a proposal $y$ and type $t_i$, $s_2^i(t_i)$ denoting the probability of voting for the incumbent legislator given voter type $t_i$ and rejection of a proposal by the legislator, and $s_3^i(t_i)$ denoting the probability of voting in favor of an initiative, given voter type $t_i$. 
Additionally, given a strategy \( s_i \), let

\[
\bar{s}_i^1(y) = \int_T s_i^1(y, t)\phi(dt)
\]

denote the *ex ante* probability that voter \( i \) votes in favor of the incumbent, conditional on the legislator enacting a proposal \( y \). Similarly, let

\[
\bar{s}_i^2 = \int_T s_i^2(t)\phi(dt)
\]

denote the *ex ante* probability that voter \( i \) votes in favor of the incumbent conditional on rejection of a proposal and

\[
\bar{s}_i^3 = \int_T s_i^3(t)\phi(dt)
\]

denote the *ex ante* probability of voting in favor of an initiative.

Finally, we denote the subjective expected payoff of \( s_i \) at information set \( I \in \mathcal{I} \), given beliefs \( \mu_i \), other voters’ strategies \( s_{-i} \), and type \( t_i \), by \( \bar{s}_i^I(s_i; \mu_i, s_{-i}, t_i) \).

### A.3 The Legislator

The action of the legislator is denoted by \( a_L \in A_L = \{0, 1\} \), where 1 represents passage of the proposal and 0 represents rejection of the proposal. The incumbent legislator seeks to maximize the number of voters who vote in favor of retaining her: \( V = \sum_{i \in N} v_i^L \). A strategy for the legislator is a function, \( \sigma : \mathbb{R}^n \rightarrow \Delta(A_L) \), from proposals to mixed strategies over \( A_L \). Formally, let \( \sigma(y) \) denote the probability that proposal \( y \) is enacted by the legislator. The legislator’s interim expected utility
upon observing proposal $y$, conditional upon strategies $s$ by the voters and $\sigma$ by the incumbent, is given by
\[ \bar{V}(\sigma, s, y) = \sigma(y) \sum_{i \in N} s_1^i(y) + (1 - \sigma(y)) \sum_{i \in N} s_2^i. \] (1)

### A.4 Equilibrium

Our equilibrium concept is *Perfect Bayesian Equilibrium* (PBE).

**Definition 1** A Perfect Bayesian Equilibrium (PBE) is a strategy profile $(\sigma^*, s^*)$ and posterior beliefs $\mu^*$ such that

\( \forall y, \sigma^*(y) \in \arg \max_{\alpha \in [0, 1]} \bar{V}(\alpha, s^*, y), \)

\( \forall i \in N, \forall I \in \mathcal{I}, s^*_i(I) \in \arg \max_{\alpha \in \Delta(A(I))} \pi^i_1(\alpha; \mu^*_i, s^*_{-i}, t_i), \) and

\( \forall i \in N, \forall t_i \in T, \forall C \in \mathcal{B}(\mathbb{R}^n), \)
\[ \mu^*_i(C, t_i) = \frac{\int_{C}(1 - \sigma^*(z))\rho(dx)}{\int_{\mathbb{R}^n}(1 - \sigma^*(x))\rho(dx)} \text{ if } \int_{\mathbb{R}^n}(1 - \sigma^*(x))\rho(dx) > 0 \text{ and } \]
\[ \mu^*_i(C) \text{ is any probability distribution if } \int_{\mathbb{R}^n}(1 - \sigma^*(x))\rho(dx) = 0. \]

The final requirement states that voters’ beliefs in subgames reached with positive probability must be consistent with the legislator’s strategy in equilibrium. The restriction of this consistency requirement to subgames which are reached with positive probability represents no real substantive restriction in our current model, as it is satisfied unless the legislator enacts $\rho$-almost all proposals (i.e., it is satisfied whenever the probability of observing initiatives (according to $\rho$) is strictly positive in equilibrium). Finally, note that since $t_i$ and $y$ are independently distributed for all $i \in N$, a voter’s type, $t_i$, does not enter into the restriction on her beliefs, $\mu^*_i(y, t_i)$, so that it must...
be the case that voters’ beliefs about \( y \) can not depend on \( t_i \) along any path of play reached with positive probability in equilibrium.\(^9\)

We restrict our attention to PBE in weakly undominated strategies. Given beliefs \( \mu_i^* \), a strategy for voter \( i \), \( s_i^* \), is a weakly undominated strategy if there exists no strategy \( s_i' \neq s_i^* \) such that for any other profile of other voters’ strategies, \( s_{-i}, \pi_i(s_i'; \mu_i^*, s_{-i}, t_i) \geq \pi_i(s_i^*; \mu_i^*, s_{-i}, t_i) \) and for some profile of other voters’ strategies, \( \bar{s}_{-i}, \pi_i(s_i'; \mu_i^*, \bar{s}_{-i}, t_i) > \pi_i(s_i^*; \mu_i^*, \bar{s}_{-i}, t_i) \). We will refer to such a PBE satisfying this requirement as an Undominated Perfect Bayesian Equilibrium.\(^10\)

**Definition 2** A vector of strategies and beliefs, \((\sigma^*, s^*, \mu^*)\), is an Undominated Perfect Bayesian Equilibrium (UPBE) if it is a perfect Bayesian equilibrium in which, for all \( i \in N \), \( s_i^* \) is weakly undominated.

**A.5 Results**

In this section, we investigate the properties of undominated perfect Bayesian equilibria of the game defined above. Our first two lemmas are presented without proof, as they follow from standard arguments. They state that, when choosing between a vote for the incumbent and a vote for the challenger, any voter using a weakly undominated voting strategy votes as if she is pivotal. Unsurprisingly, this restricts the space of possible UPBE voting strategies greatly: voters possess a unique weakly undominated action at generic information sets.

**Lemma 3** Let \((\sigma, s, \mu)\) constitute a UPBE. Then, for any \( i \in N \), \( s_i^1(y, t_i) = 1 \) if

\[
y_i + t_i^1 > 0,
\]

while \( s_i^1(y, t_i) = 0 \) if
\[ y_i + t^1_i < 0. \]

The logic behind Lemma 3 is simple: if a given voter would prefer to have the incumbent win and keep the imposed policy, her weakly undominated voting strategy in the legislative election following imposition of the policy is to vote for the incumbent.

**Lemma 4** Let \((\sigma, s, \mu)\) constitute a UPBE. Then, for any \(i \in N\), \(s^2_i(t_i) = 1\) if

\[ t^1_i > 0, \]

while \(s^2_i(t_i) = 0\) if

\[ t^1_i < 0. \]

The logic behind Lemma 4 is also straightforward: any given voter’s preferences in the legislative election are independent of the outcome of the initiative vote, so her weakly undominated voting strategy in the legislative election following rejection of the proposed policy is to vote for the incumbent if her private value from a victory by the incumbent, \(t^1_i\), is positive.

By assuming that voters vote in favor of the incumbent when indifferent between the incumbent and the challenger and in favor of the initiative when indifferent between its passage and rejection, Lemma 3 and Lemma 4 allow us to uniquely characterize voter behavior as a function of \(\phi\), the distribution of types. Accordingly, we now define \(F\) as

\[ F(z) = \Pr_{\phi}\{t_i : t^1_i \geq z\}. \]

Lemma 3 and Lemma 4 then imply that

\[ \bar{s}^1_i(y) = F(-y_i) \text{ and } \bar{s}^2_i = F(0). \]
so that we can now rewrite the legislator’s expected utility, Equation 1, in any UPBE as

\[
\bar{V}(\sigma, s, y) = \sigma(y) \sum_{i \in N} F(-y_i) + (1 - \sigma(y)) \sum_{i \in N} F(0).
\] (2)

Note that \( F \) is a strictly decreasing function bounded between 0 and 1 because it is equal to one minus the cdf of an atomless random variable with full support.

### A.6 Analysis

Let \( y_G \) denote \( y_i \) for all \( i \leq G \) and \( y_{G^c} \) denote \( y_k \) for all \( k > G \). Thus, \( y_G \) is the level of benefits for group members and \( y_{G^c} \) the level of benefits for nonmembers. Recall that we assume that \( y_G > 0 \) since, otherwise, the group would not propose the proposal to the legislator or sponsor it as an initiative. The expected utility of the legislator, as given in Equation 2, can be written as

\[
\bar{V}(\sigma, s, y) = \sigma(y) \left[ GF(-y_G) + (N - G)F(-y_{G^c}) \right] + (1 - \sigma(y)) \sum_{i \in N} F(0).
\] (3)

The legislature strictly prefers to enact a proposal \( y \) (i.e., \( \sigma(y) = 1 \)) if

\[
GF(-y_G) + (N - G)F(-y_{G^c}) > NF(0).
\]

The legislator is indifferent between enacting and rejecting any proposal \( y^* \) such that

\[
GF(-y_G^*) + (N - G)F(-y_{G^c}^*) = NF(0).
\]
We assume that the legislator rejects a proposal when indifferent so that, in any UPBE, \( \sigma(y) = 0 \) for any \( y \) such that

\[
GF(-y_G) + (N - G)F(-y_{GC}) \leq NF(0). \tag{4}
\]

We denote the size of the group at which the legislator is indifferent between accepting and rejecting any given proposal \( y \) by \( G^*(y) \):

\[
G^*(y_G, y_{GC}) = N \frac{F(0) - F(-y_{GC})}{F(-y_G) - F(-y_{GC})}, \tag{5}
\]

Since \( \phi \) is a continuous probability measure, \( G^*(y) \) is a continuous function of all of its arguments for \( y_G \neq y_{GC} \). The first derivative of \( G^* \) with respect to \( y_{GC} \) is

\[
\frac{dG^*}{dy_{GC}} = -N \frac{dF(-y_{GC})}{dy_{GC}} \left[ \frac{F(0) - F(-y_G)}{[F(-y_G) - F(-y_{GC})]^2} \right].
\]

We now note the following two facts, which follow from the fact that \( F \) is a strictly decreasing function, and the only proposals of interest are \( y \in Y \) such that \( y_G > 0 > y_{GC} \):

\[
\frac{dF(-y_{GC})}{dy_{GC}} > 0
\]

and

\[
(y_G > 0) \Rightarrow F(0) - F(-y_G) < 0,
\]

which jointly imply that

\[
\frac{dG^*}{dy_{GC}} < 0.
\]
Thus, group size and minimum acceptable utility offered to nonmembers are inversely related. We now define the following function (since $G^*$ is a continuous function of $y_{GC}$ so long as $y_G \neq y_{GC}$), which defines the level of nonmember benefits which make the legislator indifferent between accepting and rejecting a proposal, given the level of member benefits and the size of the group, $y^*_C(y_G, G)$:

$$y^*_C(y_G, G) = -(F)^{-1} \left[ \frac{NF(0) - GF(-y_G)}{N - G} \right],$$

where $-(F)^{-1}$ denotes the inverse of $F$ (which we know exists due to the fact that $F$ is a strictly decreasing function). It follows from the fact that $G^*$ is a decreasing function of $y_{GC}$ for the proposals of interest that $y^*_C$ is also a decreasing function of $G$. In addition, it is clear that $y^*_C$ is a decreasing function of $y_G$ whenever $G > 0$.

Defining the set $Y^* = \{y \in Y : y_G < y^*_C(y_G, G)\}$, our next result states that $Y^*$ is the set of proposals that the legislator does not strictly prefer to enact and that the interior of this set is the set of proposals that the legislator strictly prefers to reject.

**Proposition 5** In any UPBE, $(\sigma, s, \mu)$, $y \notin Y^*$ implies that $\sigma(y) = 1$. Furthermore, $y \in \text{Int}(Y^*)$ implies that $\sigma(y) = 0$.

**Proof:** This follows immediately from Lemma 3, Lemma 4, and Equation 4.

Now define the set $Y^{**} = \{y \in Y : y_G > 0, y_{GC} \leq y^*_C(y_G, G)\}$ to be the set of proposals which will be proposed by the group but rejected by the legislator - the set of proposals which will be observed as initiatives in any UPBE in which the legislator rejects a proposal whenever she is indifferent.
Proposition 6 In any UPBE, \((\sigma, s, \mu)\), the expected utility of an initiative for a nonmember of the group is a decreasing function of the group size, \(G\).

Proof: In any UPBE, for any \(i \in N\), all \(t_i \in T\), and all \(y \in Y\), \(\mu_i\) satisfies

\[
\mu_i(y, t_i) = \frac{\rho(y)(1 - \sigma(y))}{\int_{\mathbb{R}^n}(1 - \sigma(x))\rho(dx)}.
\]

We are interested in a voter’s inference concerning \(y_i\) conditional on rejection of a proposal by the legislature which, for a nonmember, is given by:

\[
E_{\mu_i(y, t_i)}[y_{GC}] = \int_{Y}^{*} y_{GC} \frac{\rho(y)(1 - \sigma(y))}{\int_{\mathbb{R}^n}(1 - \sigma(x))\rho(dx)} dy,
\]

or

\[
E_{\mu_i(y, t_i)}[y_{GC}] = \int_{Y}^{*} y_{GC} \mathbf{1}[y_{GC} \leq y_{GC}^*(y_G, G)] \frac{\rho(y)(1 - \sigma(y))}{\int_{\mathbb{R}^n}(1 - \sigma(x))\rho(dx)} dy,
\]

where the first term in the integration in Equation 6 results from the fact that the legislature accepts any proposal \(y\) with \(y_{GC} > y_{GC}^*(y_G, G)\), implying that \(1 - \sigma(y) = 0\) for any such \(y\). Of course, when \(G\) changes, then \(\mu\), the equilibrium beliefs, must change as well. However, by Proposition 5, \(\mu\) represents the conditional probability of \(y\) such that \(y \in Y^*\). Therefore, \(E_{\mu_i(y, t_i)}[y_{GC}]\) is simply the expected value of \(y_{GC}\), given \(\rho\) and conditional on \(y_{GC} < y_{GC}^*\). Since \(y_{GC}^*\) is a decreasing function of \(G\) for all possible values of \(Y_G > 0\), then this conditional expected value, as defined in Equation 6, also must be a decreasing function of \(G\). In fact, our result is stronger: the voters know that, in equilibrium, regardless of what the unknown value of \(y_G\) is for an initiative, larger values of \(G\) imply that the distribution of \(y_{GC}\) is worse in the sense of first order stochastic dominance. □
The final proposition deals with a counterfactual. Suppose that a nonmember learned the value of the proposal to the group, $y_G$. How would this affect the nonmember’s inference concerning $y_{GC}$? The next proposition states that the expected value of $y_{GC}$ is a decreasing function of $y_G$.

**Proposition 7**  In any UPBE, $(\sigma, s, \mu)$, the conditional expected utility of an initiative for a non-member of the group is a decreasing function of the utility offered to a member, $y_G$.

*Proof:* As in the proof of Proposition 6, if $(\sigma, s, \mu)$ is a UPBE, then beliefs satisfy Equation 6. Since Equation 6 is simply the conditional expected value of $y_{GC} \leq y_{GC}^*$ and $y_{GC}^*$ is a decreasing function of $y_G$, then so is Equation 6.
Figure 2: Frequency of Outcomes as a Function of Group Size
Figure 3: Expected Utilities from Initiatives
Figure 4: Average Utilities when Incumbent Enacts Proposal
Figure 5: Correlations Between Member and Nonmember Utilities As A Function of Group Size